Warsaw University of Technology

Faculty of Power and Aeronautical Engineering
WARSAW UNIVERSITY OF TECHNOLOGY

Institute of Aeronautics and Applied Mechanics

Finite element method 2 (FEM2)

Error estimation in a finite element

10.2021

A-POSTERIORI ERROR APPROXIMATION

VECTOR OF THE AVERAGE STRESS AT NODE i :

$$\{\sigma\}_i^{ave} = \frac{\sum_{e=1}^{k} \{\sigma\}_i^e}{k}$$

where :

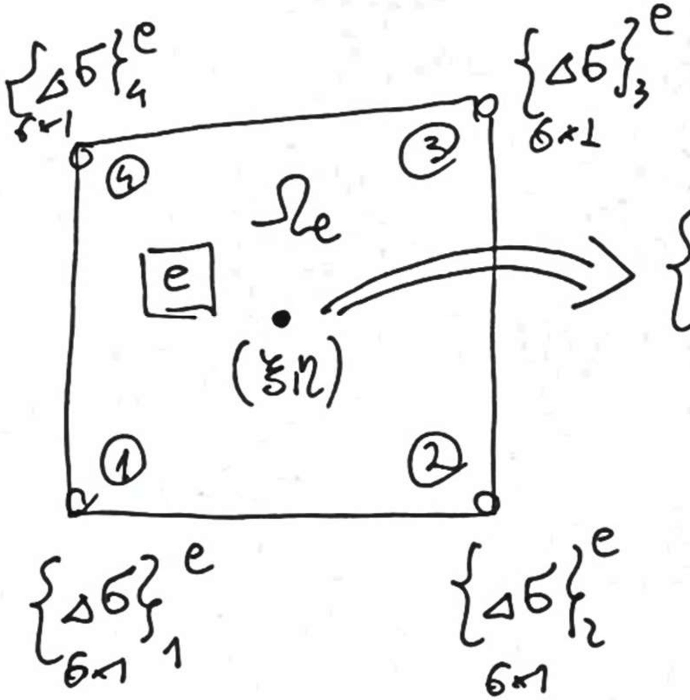
k – number of elements adjacent to node i

e – element number

$$\underset{6\times 1}{\{\sigma\}_i^e} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}_i^e$$

THE STRESS ERROR IN ELEMENT e AT NODE i :

$$\underset{6\times1}{\{\Delta\sigma\}_i^e} = \underset{6\times1}{\{\sigma\}_i^e} - \{\sigma\}_i^{ave}$$



$$\underset{6\times1}{\{\Delta\sigma(\xi,\eta)\}^e} = \sum_{i=1}^{4} \{\Delta\sigma\}_i^e \cdot N_i(\xi,\eta)$$

The maximum absolute value of any stress component :

$$\max\left(\underset{6\times1}{\{\Delta\sigma\}_i^{e}}\right) \qquad (\text{SDSG- IN ANSYS})$$

Strain energy in a finite element:

$$U_e = \frac{1}{2}\int_{\Omega_e} \underset{1\times6}{\lfloor\sigma\rfloor^{e}}\cdot\underset{6\times1}{\{\varepsilon\}^{e}}\, d\Omega_e = \frac{1}{2}\int_{\Omega_e} \underset{1\times6}{\lfloor\sigma\rfloor^{e}}\,\underset{\underset{\text{Inverse constitutive matrix}}{\uparrow}}{\underset{6\times6}{[D]^{-1}}}\,\underset{6\times1}{\{\sigma\}^{e}}\, d\Omega_e$$

strain energy density in a finite element

$$u_e = \frac{U_e}{\int_{\Omega_e} d\Omega_e} \qquad (\text{SENDELAS – IN ANSYS})$$

Total strain energy for the entire model:

$$U = \sum_{e=1}^{NOE} U_e$$

strain energy error in a finite element:

$$\Delta U_e = \frac{1}{2} \int_{\Omega_e} \underbrace{\lfloor \Delta \sigma \rfloor^e}_{1\times 6} \underbrace{[D]^{-1}}_{6\times 6} \cdot \underbrace{\{\Delta \sigma\}^e}_{6\times 1} \, d\Omega_e$$

(SERR-IN ANSYS)

Total strain energy error in the entire model:

$$\Delta U = \sum_{e=1}^{NOE} \Delta U_e$$

The normalized energy error (for the entire model) is calculated as:

$$S = 100 \cdot \sqrt{\frac{\Delta U}{\Delta U + U}}$$

(SEPC IN ANSYS)

The procedure of mesh refining:

$\Delta U_e = \text{const}$ for all FEs – „error equilibration"

$S < S_0$ for the entire model

S_0 – allovable value of S.

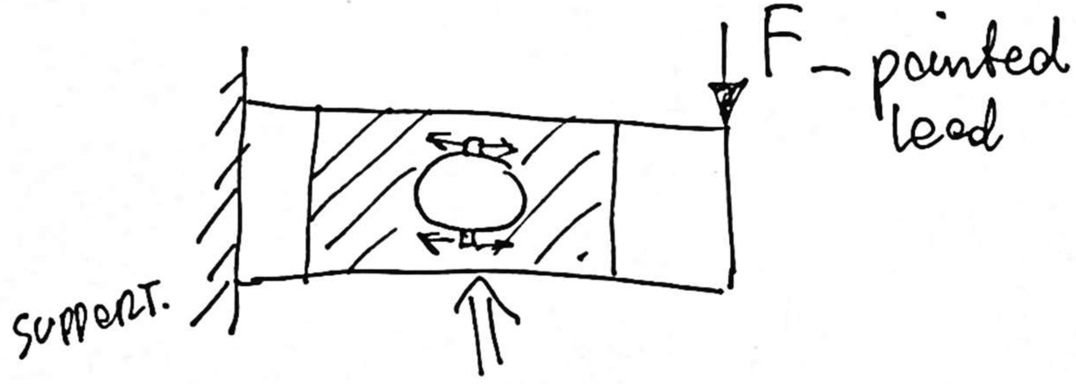
1°) Adaptive meshing – to correct mesh until:

$$\frac{\Delta U_e - \frac{\Delta U}{NOE}}{\frac{\Delta U}{NOE}} < \Delta U_0$$

↑ allovable value.

and $S < S_0$

2°) Selective adaptive meshing



TYPES OF REFINEMENT:

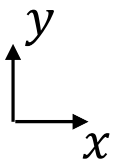
h – size decrease

p – polynomial order in shape functions

r – nodes rearrangement

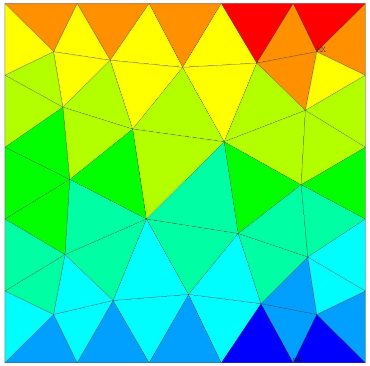
h_p – combination of „h" and „p"

Normal stress in x direction (SX)



CST, 4mm

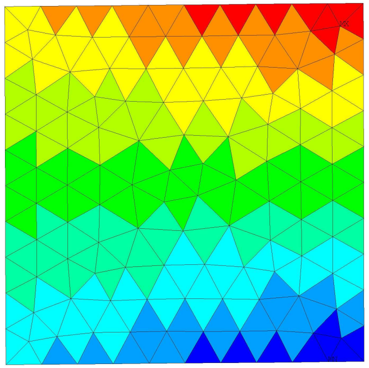
(free mesh)



SMN =29.9391
SMX =50.0735

CST, 2mm

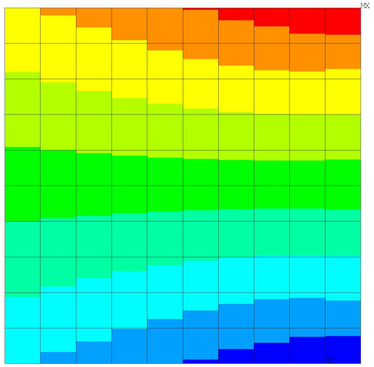
(free mesh)



SMN =31.3547
SMX =48.657

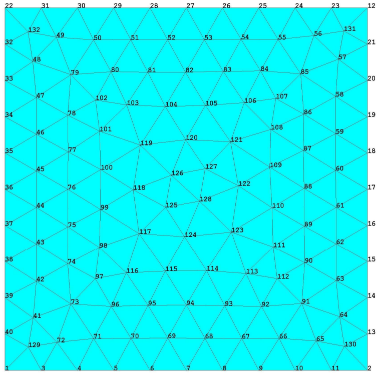
QUAD-4node, 2mm

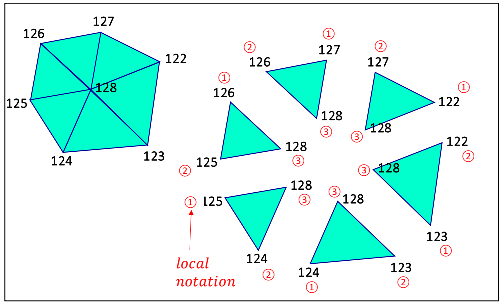
(mapped mesh)



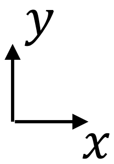
SMN =31.2497
SMX =48.819

FEM 1. HOMEWORK 1

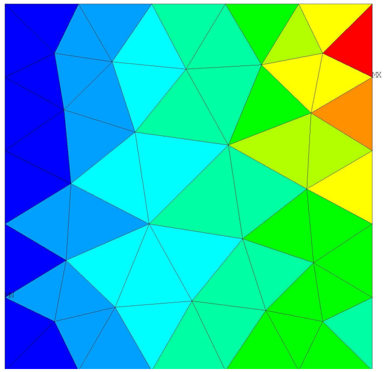




Normal stress in y direction (SY)

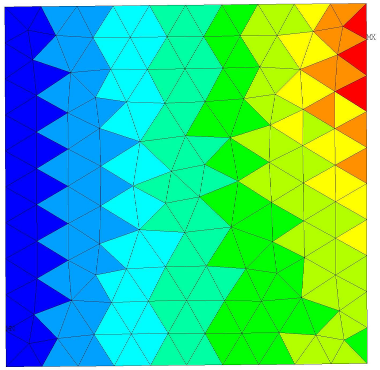


CST, 4mm



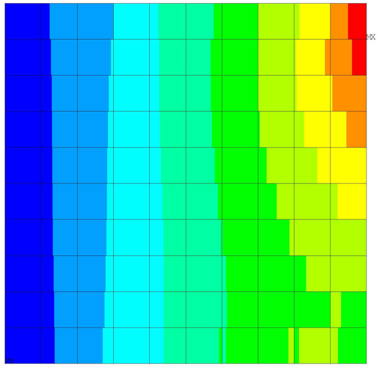
SMN =22.4161
SMX =31.2216

CST, 2mm



SMN =22.4077
SMX =30.4297

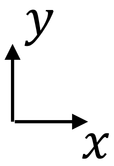
QUAD-4node, 2mm



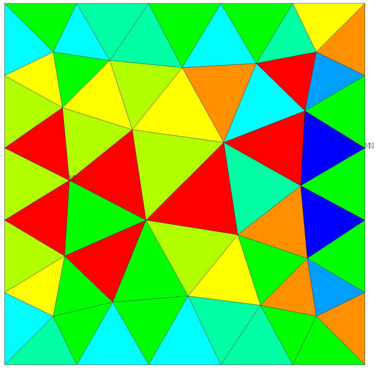
SMN =22.3724
SMX =30.5656

(MPa)

Shear stress (SXY)

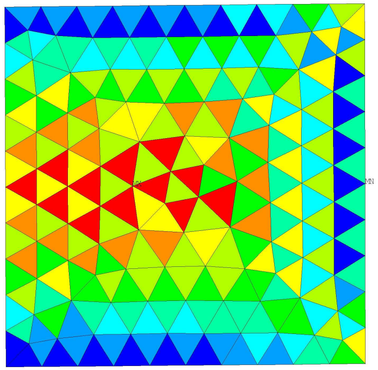


CST, 4mm



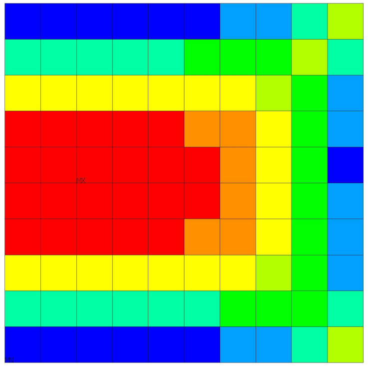
SMN =20.9712
SMX =22.9216

CST, 2mm



SMN =21.4276
SMX =22.7416

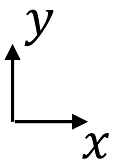
QUAD-4node, 2mm



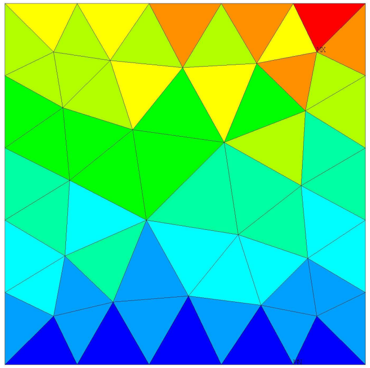
SMN =21.553
SMX =22.4885

(MPa)

Von Mises stress (SEQV)

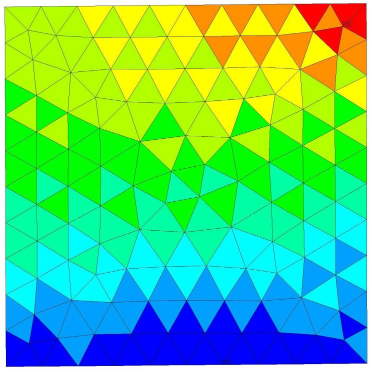


CST, 4mm



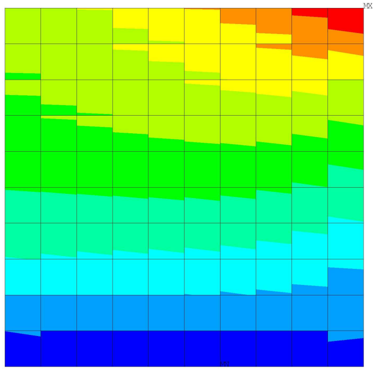
SMN =47.474
SMX =58.2418

CST, 2mm



SMN =47.9007
SMX =57.2223

QUAD-4node, 2mm



SMN =47.9151
SMX =57.431

(MPa)

Elastic strain energy (SENE)

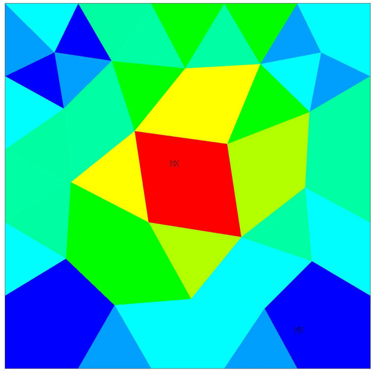
$$U_e = \frac{1}{2}\int_{\Omega_e} \underset{1\times 6}{\lfloor \varepsilon \rfloor}\underset{6\times 1}{\{\sigma\}}\, d\Omega_e$$ (Nmm)

CST, 4mm

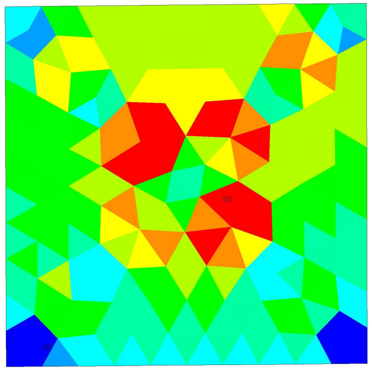
CST, 2mm

QUAD-4node, 2mm

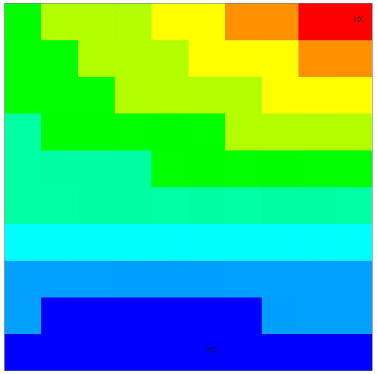
(SENE)



SMN =.029352
SMX =.093964

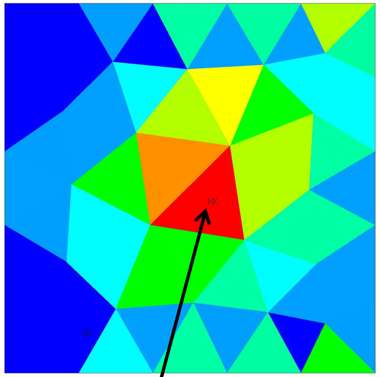


SMN =.007026
SMX =.019023

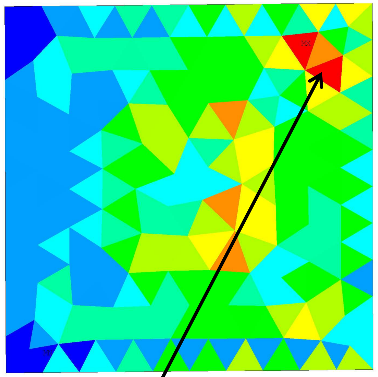


SMN =.024947
SMX =.035708

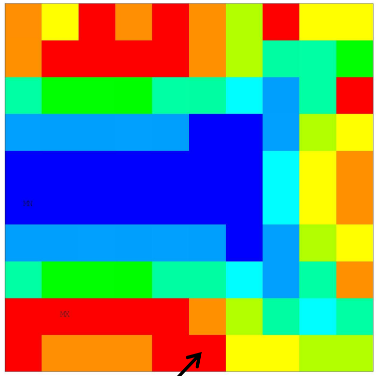
Error (SERR)



SMN =.944E-05
SMX =.155E-03
(SEPC= 4.06%)



SMN =.434E-06
SMX =.624E-05
(SEPC= 1.92%)

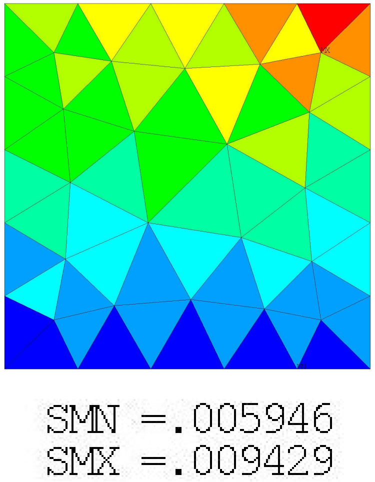


SMN =.309E-07
SMX =.945E-06
(SEPC= 0.51%)

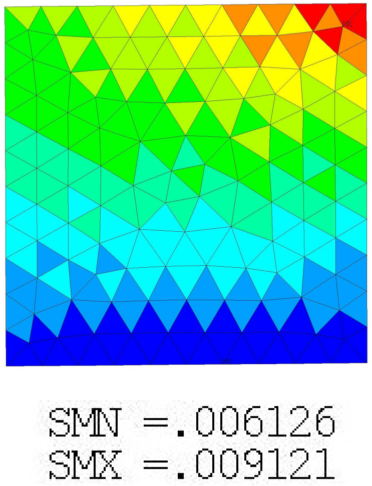
Elastic strain energy density (SENDELAS)

$$u_e = \frac{U_e}{\int_{\Omega_e} d\Omega_e}$$

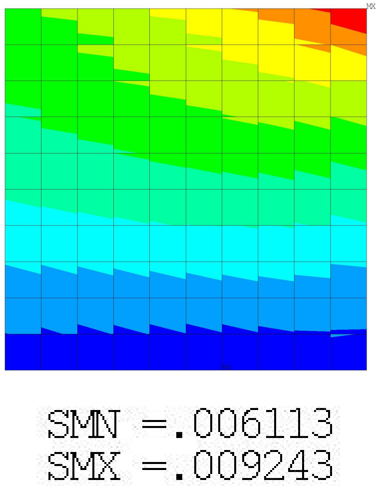
CST, 4mm



CST, 2mm



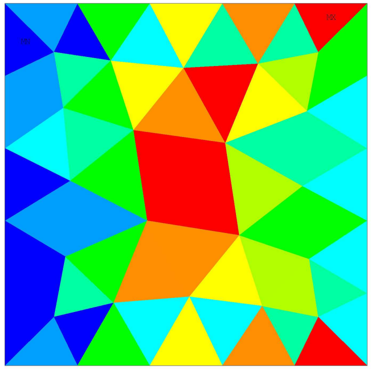
QUAD-4node, 2mm



(Nmm/mm^3)

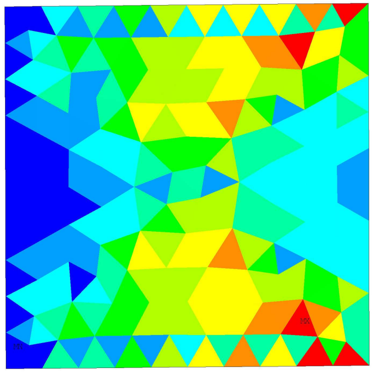
Maximum absolute value of error of any stress component (SDSG)

CST, 4mm



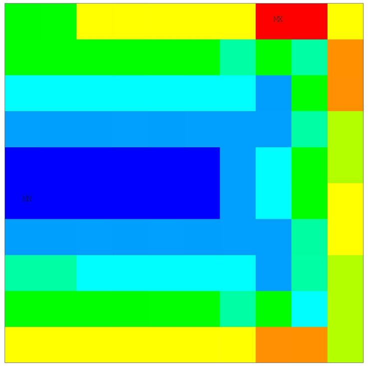
SMN =1.07305
SMX =3.37362

CST, 2mm



SMN =.490022
SMX =1.61018

QUAD-4node, 2mm



SMN =.045321
SMX =.338378

(MPa)